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Technical Notes

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Effect of Inlet Conditions on the Pressure Drop in Confined Vortex Flow

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I. Introduction

CONFINED swirling flows find a wide range of applications in practical devices and industrial equipment such as in vortex separators, gas turbine combustors, furnaces, and combustors. Many of these applications are summarized by Gupta et al.[1] Two interesting features of the flow of practical importance are the pressure drop and the viscous core size. The pressure drop across a vortex chamber has been the subject of several papers; for example, see Shakespeare and Levy,[2] Escudier et al.,[3] and Jawarneh et al.[4] These studies have shown that the inlet conditions significantly influence the main flow. Our previous work has focused on chambers with a single vortex generator. The pressure coefficient has been shown to be a function of the chamber geometry and the Reynolds number. In this Note the same properties will be examined for a double-vortex generator chamber.

II. Experiments

The present experiments have been conducted using a jet-driven vortex chamber similar to the one used by Vatistas et al.[5] The main difference between the two is that in the latest version, shown schematically in Fig. 1, it has a cylindrical configuration with constant cross-sectional area ($R_o = 7$ cm) and a central axis outlet and circumferential inlets. Swirl is imparted to the fluid by the two vortex generators shown in Fig. 1. Each has four radial inlets where the compressed air is introduced. The required set of inlet conditions is obtained by inserting the appropriate vortex generator blocks (swirler) into the vortex generator assembly. A number of openings of a circular cross section d_{in} are drilled at an angle of $\varphi = 30$ deg. When airflow passes through the swirlers, it is guided into the vortex chamber by the tangential inlets producing the required swirl inside the chamber. The two counterclockwise generators are mounted at the two ends of the vortex chamber. Each generator has 16 holes with diameter $d_{\text{in}} = 1.267$ cm and inlet area ($A_{\text{in}_{1,2}} = 20.177$ cm^2). For the experiments reported here, the chamber length is $L = 42$ cm, and the diameter of the exit hole ($2R_e$) varies from 1.88 to 4.2 cm. The static pressure is measured by a series of taps located a head of the tangential ports and is averaged by connecting in parallel all of the pressure pick-up tubes into a common tube. The measurements of the mean gauge pressure ($P_{\text{in}} - P_a$) were obtained using a U-tube filled with Meriam oil, having a specific gravity equal to 1.00. The measurements were made at two inlet airflow rates Q_{in}: 0.0117 and 0.0187 m^3/s, which correspond to two Reynolds numbers (R_{eo}): 7.245×10^3 and 11.592×10^3. The estimated uncertainty for the pressure drop measurements is between 8–10%. A rotameter was used to measure the volumetric flow rate of the inlet air. This was calibrated in standard conditions (1 atm and 20°C). For the flow rates used, the uncertainty was estimated to be from 1.4 to 2.0%.

III. Analysis

It has been shown previously[4] for a chamber with a single vortex generator that the pressure drop coefficient is given by

$$C_p = \frac{\alpha^2\xi^4}{(1-\chi^2)^2} - 2\frac{\delta^2\xi^2\cos^2(\varphi)\,\ell n(\chi)}{(1-\chi^2)} - 2\frac{(1-\delta^2)\cos^2(\varphi)\,\ell n(x/\xi)}{1-(\chi/\xi)^2} - 1 \tag{1}$$

where

$$C_p = 2\Delta p \big/ \rho q_{\text{in}}^2, \qquad \Delta p = p_{\text{in}} - p_a, \qquad \chi = R_c/R_e$$
$$\xi = R_o/R_e, \qquad \varphi = \cos^{-1}(V_{\varphi\text{in}}/q_{\text{in}})$$
$$\zeta = L/D_o, \qquad \alpha = A_{\text{in}}/A_o$$

and the rest of the parameters P_a, P_{in}, q_{in}, $V_{\varphi\text{in}}$, A_{in_1}, A_o, α, χ, ζ, ξ are ambient static pressure, static pressure at the inlet, total velocity vector at the inlet, inlet tangential velocity component, total inlet area of a single vortex generator, cross-sectional area of the vortex chamber, area ratio, dimensionless core size at exit, aspect ratio, and diameter ratio, respectively.

The following theortical formulation of the vortex chamber with double-vortex generators is an extension to our previous studies that dealt with a single generator. The energy equation for a strongly swirling flow will be considered here. To simplify the problem, several assumptions have been made. These are the pressure and the total velocity through the inlet ports of the two vortex generators are both uniform; the radial velocity at the exit is neglected because it does not have the space and time to develop; and at the exit the pressure is ambient. Energy balance over the control volume enclosing

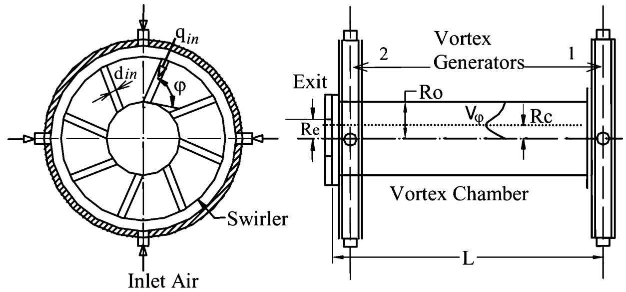

Fig. 1 Schematic of the vortex chamber with double-vortex generator.

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the chamber and the double-vortex generators yields

$$\left[\frac{p_{\mathrm{in}_1}}{\rho}+\frac{1}{2}q_{\mathrm{in}_1}^2\right]Q_{\mathrm{in}_1}+\left[\frac{p_{\mathrm{in}_2}}{\rho}+\frac{1}{2}q_{\mathrm{in}_2}^2\right]Q_{\mathrm{in}_2} = \int_{R_c}^{R_e}\left[\frac{p_a}{\rho}+\frac{1}{2}\left(V_{\varphi\,\mathrm{out}}^2+V_{z\,\mathrm{out}}^2\right)\right]V_{z\,\mathrm{out}}\,2\pi r\,\mathrm{d}r \tag{2}$$

where the subscripts 1 and 2 refer to the inlet flow for the first and the second generator, respectively. The two generators have the same geometry ($A_{\mathrm{in}_1}=A_{\mathrm{in}_2}$, $\varphi_1=\varphi_2$), and the same inlet velocity and flow rates ($q_{\mathrm{in}}=q_{\mathrm{in}_1}=q_{\mathrm{in}_2}$, $Q_{\mathrm{in}_1}=Q_{\mathrm{in}_2}$), and so then the inlet static pressure must be equal for both generators ($P_{\mathrm{in}_1}=P_{\mathrm{in}_2}$). The $n=2$ Vatistas et al.[6] vortex model will be used, whereby the tangential velocity is given by

$$V_\varphi = V_{c\,\mathrm{in}}R_c\left[r\Big/\left(R_c^4+r^4\right)^{\frac{1}{2}}\right] \qquad \text{where} \qquad V_{c\,\mathrm{in}}=\Gamma/2\pi R_c$$
$$\Gamma = V_{\varphi\,\mathrm{in}}2\pi R_o \qquad \text{and} \qquad V_{\varphi\,\mathrm{in}}=q_{\mathrm{in}}\cos(\varphi)$$

where Γ and R_c are the vortex circulation and the core radius, respectively. The essence of the present analysis lies in the application of energy equation, where the bulk of the loss is assumed to occur across the vortex chamber. The vortex decay factor δ is expected to vary with the Reynolds number, swirl angle, and area ratio. It is evident from the experimental results of Yan et al.[7] that the vortex decay factor value is between zero and one. Mathematically, it can be given by

$$\delta = V_{c\,\mathrm{out}}/V_{c\,\mathrm{in}}$$

where $V_{c\,\mathrm{in}}$, $V_{c\,\mathrm{out}}$ are the vortex strength at the inlet and outlet, respectively. The outlet swirl velocity at the vortex chamber exit is given by

$$V_{\varphi\,\mathrm{out}} = V_{c\,\mathrm{out}}R_c\left[r\Big/\left(R_c^4+r^4\right)^{\frac{1}{2}}\right]$$

Average axial outlet velocity is assumed at the exit port of the chamber,

$$V_{z\,\mathrm{out}} = Q_{\mathrm{out}}\Big/\pi\left(R_e^2-R_c^2\right)$$

From mass conservation, $Q_{\mathrm{in}_1}+Q_{\mathrm{in}_2}=Q_{\mathrm{in}}=Q_{\mathrm{out}}$, and $Q_{\mathrm{in}}=q_{\mathrm{in}}A_{\mathrm{in}}$, $A_{\mathrm{in}}=2A_{\mathrm{in}_1}$.

After performing the integration of Eq. (2), then the following equation for the pressure drop coefficient C_p is obtained:

$$C_p = \delta^2\cos^2(\varphi)\xi^2\frac{\ell n\left[\frac{1}{2}(1+1/\chi^4)\right]}{2(1-\chi^2)}+4\alpha^2\xi^4\frac{1}{(1-\chi^2)^2}-1 \tag{3}$$

and the Reynolds number R_{eo} based on the average bulk velocity is defined as

$$R_{\mathrm{eo}} = 4Q_{\mathrm{in}}/\nu\pi D_o$$

At a given set of design geometric parameters ξ, φ, α and a vortex decay factor δ, then

$$C_p = f_n(\chi)$$

Equation (3) reveals that C_p is unbounded when χ tends to zero or one; therefore, there must exist $0<\chi<1$ such that C_p is minimum. The latter required that the minimum C_p principle yields

$$\frac{\mathrm{d}C_p}{\mathrm{d}\chi}=0$$

or

$$\delta^2\cos^2(\varphi)\left\{\ell n\left[0.5\left(1+\frac{1}{\chi^4}\right)\right]-\frac{2(1-\chi^2)}{\chi^2(1+\chi^4)}\right\} + 16\alpha^2\xi^2\frac{1}{(1-\chi^2)}=0.0 \tag{4}$$

Given the values of vortex chamber geometries (ξ, φ, α) and δ, Eq. (4) can be solved numerically for χ using any of traditional root finding methods. To know the value of δ, one must know how the vortex decays. Its value will be found based on the experimental results using a modified version of the least-squares technique. The observations will provide the data of C_p across a vortex chamber operating under specific conditions. The preceding theory will then be applied to curve fit the results assuming values of δ and calculating the square error according to the formula:

$$E = \sum_{i=1}^{N}(C_{p\,\mathrm{exp}\,i}-C_{p\,\mathrm{theor}\,i})^2$$

Then the optimum δ for a given set of data will then be the one that produces the least-square error E.

IV. Analysis of Results

In Fig. 2 the present experimental data are compared with the current theory in terms of the pressure drop coefficient C_p for aspect ratio $\zeta=3.0$ and an inlet angle $\varphi=30$ deg. In addition, the same parameters are contrasted with those obtained using a chamber with a single generator.[4] It is clear that as the diameter ratio ξ and the Reynolds number R_{eo} increase, the pressure coefficient C_p increases for a single and double generators. Stronger vortices will be produced by increasing the diameter ratio and/or Reynolds number,

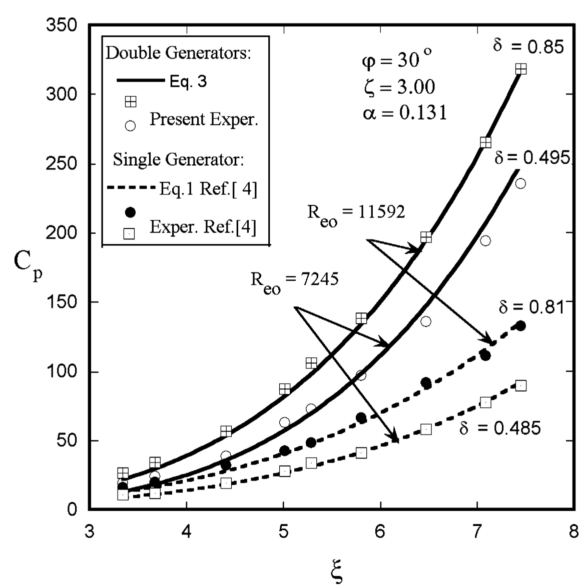

Fig. 2 Pressure drop coefficient for single- and double-vortex generator chamber at different Reynolds numbers.

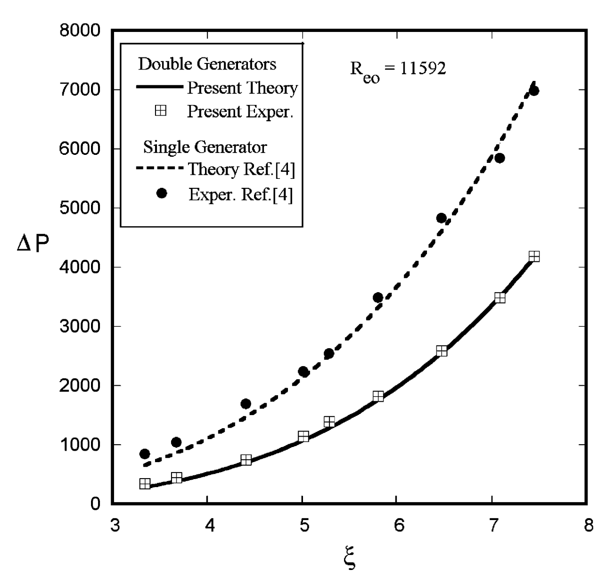

Fig. 3 Actual pressure drop for single- and double-vortex generator chamber at $R_{\mathrm{eo}}=11.592\times10^3$.

resulting in a higher tangential velocity and hence a higher pressure drop. At specific diameter ratio ξ and R_{eo}, the pressure coefficient C_p for double generators is higher than of a single generator, but the actual pressure drop ($\Delta p = P_{in} - P_a$) is less (see Fig. 3). Because the total inlet area A_{in} increases and the total inlet velocity q_{in} slows down, then the vortex strength will be weakened. The percentage reduction for example at $\xi = 6$ is 46%. Therefore, more energy can be saved by using two generators than one generator.

V. Conclusions

The present study explored the effects of two kinds of vortex generators on the pressure drop. It has been seen that the pressure drop coefficient increases with increasing the Reynolds number and/or the diameter ratio. A stronger vortex will be produced by increasing the diameter ratio and/or Reynolds number, resulting in a higher tangential velocity and hence a higher pressure drop. The actual pressure drop is less for the double generators than of a single generator. Therefore, energy can be saved by using two generators instead of one.

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